

PARAMETRIC YIELD CURVE MODELING IN AN ILLIQUID AND UNDEVELOPED FINANCIAL MARKET

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Abstract:

This paper examines the possibility of applying two most popular parametric yield curve models (Nelson-Siegel and Svensson) in the Croatian financial market. In such an illiquid and undeveloped financial market yield curve modeling presents a special challenge primarily regarding the available market data. The use of the yield curve models is limited compared to the developed markets and the interpretation of the resulting yield curves requires much more cautiousness. However this paper clearly shows that the yield curve model is able to capture changes in the business cycle according to the macroeconomic theory and therefore provide valuable information to the financial industry and other economic subjects. It also suggests that the Svensson model which is an extension of the Nelson-Siegel model (and is therefore often preferred over the Nelson-Siegel model in the developed markets) suffers from overparameterization in the illiquid and undeveloped Croatian financial market.

Keywords: parametric yield curve models, illiquidity.

Jel Classification: G12, E43, E44

INTRODUCTION

The yield curve plays a crucial role in the modern financial markets. A wide body of literature has therefore appeared since 1970's concerning the yield curve modeling. Most of the research (especially during the 20th century) has reasonably been more or less directly related to the most developed financial markets in the world. More recently a significant portion of research dedicated to yield curve modeling in the illiquid and undeveloped financial markets has emerged wherever such conditions in the financial markets may appear. In the past decade papers have been published regarding yield curve modeling in the financial markets of India, China, Taiwan,

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Russia, Slovenia and Serbia among others. Liquidity issues have been specifically addressed in the work of Dutta, Basu, and Vaidyanathan (2005), Chou et al. (2009) and Smirnov and Zakharov (2003) concerning the Indian, Taiwanese and Russian financial markets respectively. In the mentioned papers various yield curve modeling approaches were considered and empirically tested including the parametric Nelson-Siegel and Svensson models. Table 1. below summarizes key research findings of the selected papers for the above mentioned financial markets.

Table 1. Overview of the selected yield curve modeling research in the illiquid and undeveloped financial markets

Market	Research	Analyzed models	Preferred model
Indian	Virmani (2006)	- Nelson-Siegel and Svensson - CIR	Svensson and CIR equally good
	Dutta et al. (2005)	- Svensson - B spline and smoothing spline	Svensson
	Subramanian (2001)	- Nelson-Siegel and Svensson - Cubic B spline and smoothing spline	Svensson
	Nag and Ghose (2000)	- Nelson-Siegel	-
Chinese	Xie et al. (2006)	- Nelson-Siegel and Svensson - Cubic spline, exponential spline and 3 B spline models (linear, exponential and integrated)	Exponential spline
Taiwanese	Chou et al. (2009)	- Nelson-Siegel and Svensson	Svensson
Slovenian	Grum (2006)	- Nelson-Siegel and Svensson - B spline, smoothing B spline and - Merrill Lynch exponential spline	Nelson-Siegel

Source: Zoricic 2012, according to the research stated

Table 1. seems to suggest that parametric (Nelson-Siegel and Svensson) models dominate spline and other yield curve modeling approaches in illiquid and undeveloped financial markets. Furthermore it should be noted that in the case of the smallest financial market in the table, the Slovenian one, Nelson-Siegel model is preferred rather than the Svensson model. This is contrary to research concerning the Indian and Taiwanese financial markets. Regarding this issue it should also be mentioned that papers referring to probably the least developed financial market — Serbian financial market, continuously used the Nelson Siegel model exclusively. Papers published by the Jefferson Institute (2005), Drenovak (2006) and Zdravkovic (2010) modeled the yield curve relying only on the Nelson Siegel model². Since the Croatian financial market is much more similar to Slovenian and Serbian financial markets in any aspect than the other markets in Table 1. it is likely to expect that the Nelson-Siegel could be preferred over the Svensson model in the Croatian financial market too. For analysis regarding the size and liquidity of the Croatian financial market refer to Bogdan et al. (2012) and Zoricic (2012).

² Therefore they were omitted in the Table 1.

1. METHODOLOGY

Parametric yield curve modeling is based on a single-piece polynomial function³ for the whole maturity spectrum of yields (Choudhry 2004, 104). The parametric approach is therefore usually characterized by parsimony and the fact that the estimated parameters make economic sense (Martellini et al. 2003, 116). The most popular parametric model is the Nelson-Siegel yield curve model. Its popularity can mostly be attributed to its relative simplicity which does not affect the model's ability to fit the market data reasonably well. The same applies to other parametric models based on the Nelson-Siegel model such as Svensson (1994), Wiseman (1994) and Cairns model (1998). The before mentioned models therefore belong to a class of Nelson-Siegel models. In this research Nelson-Siegel and Svensson models will be tested as the most popular parametric models⁴ even in the case of illiquid and undeveloped financial markets as suggested by the Table 1.

1.1. Estimation of the Nelson-Siegel yield curve model parameters

Nelson and Siegel (1987) have proposed a parsimonious yield curve model that is flexible enough to produce upward sloping, downward sloping and humped yield curves. Although it rests only on estimation of four parameters the model is able to capture the most common shapes the yield curve takes on in practice. In their research the authors approach the yield curve modeling by defining the forward yield curve in order to ensure that the resulting forward curve is smooth and asymptotic which are desirable properties from theoretical standpoint (Grum 2006, 62). They present the following equation (Nelson and Siegel 1987, 475):

$$f(n, \beta) = \beta_1 + \beta_2 e^{\left(\frac{-n}{\lambda}\right)} + \beta_3 \frac{n}{\lambda} e^{\left(\frac{-n}{\lambda}\right)} \quad (1)$$

where the $f(n, \beta)$ represents the forward yield curve function which depends on maturity n and parameters λ , β_1 , β_2 and β_3 . Parameter β_1 represents asymptote while parameters β_2 and β_3 enable estimation of various shapes of the yield curve allowed by the model. If equation (1) is integrated and divided by n a zero-coupon yield curve can be derived (zero-coupon yield is an average of the forward yields). Furthermore if the parameters β_1 , β_2 and β_3 are grouped the equation is transformed into (Zdravkovic 2010, 28):

$$Z_n = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \beta_3 \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \quad (2)$$

where Z_n represents the zero-coupon yield curve function and β parameters represent yield curve factors. Thus β_1 refers to the level, β_2 to the slope and β_3 to the curvature of

³ As opposed to piecewise polynomial functions used in the spline models (BIS 2005, 8).

⁴ There are also other well known parametric models which are omitted here as they deal with the yield curve dynamics rather than the yield curve fitting. Such models include Vasicek (1977), Longstaff Schwartz (1992), Chen (1996), Bjork and Christensen (1997) and other models. It can be mentioned that the Bjork and Christensen model also belongs to the Nelson-Siegel class of yield curve models and that the Longstaff Schwartz and Chen models are not parsimonious as they require 10 and 13 parameters to be estimated respectively.

the yield curve. Expressions in parenthesis are called factor loadings which are defined by maturity n and factor λ .

Equation (2) can be written as a regression equation taking on the functional form given by equation (3):

$$y_t(\tau) = H * \beta_t + e_t, \quad (3)$$

where $y_t(\tau)$ represents a vector of yields observed in the moment t for T maturities in the vector τ , error term is given by $e \sim N(0, R)$ and H represents factor loadings matrix (Nyholm 2008, 71):

$$H = \begin{bmatrix} 1 & \frac{1 - \exp(-\lambda\tau_1)}{\lambda\tau_1} & \frac{1 - \exp(-\lambda\tau_1)}{\lambda\tau_1} - \exp(-\lambda\tau_1) \\ 1 & \frac{1 - \exp(-\lambda\tau_2)}{\lambda\tau_2} & \frac{1 - \exp(-\lambda\tau_2)}{\lambda\tau_2} - \exp(-\lambda\tau_2) \\ \vdots & \vdots & \vdots \\ 1 & \frac{1 - \exp(-\lambda\tau_T)}{\lambda\tau_T} & \frac{1 - \exp(-\lambda\tau_T)}{\lambda\tau_T} - \exp(-\lambda\tau_T) \end{bmatrix}. \quad (4)$$

Also following the work of Nyholm (2008) it can be noted that the above stated expression can be linearized if the parameter λ is treated as a constant rather than being a variable that needs to be estimated. In such a scenario the regression equation given by (3) is solved for different values of parameter λ chosen arbitrarily from a predetermined interval. The optimal value for parameter λ is then finally determined as the one which minimizes the sum of squared residuals between the observed and yields estimated by the model (Nyholm 2008, 71–72). The described approach has been adopted and pursued in this paper.

Once estimated, the parameters of the Nelson Siegel model can be used to estimate the whole term structure (i.e. estimate any yield for any given maturity) for any given point in time in the analyzed sample.

1.2. Estimation of the Svensson yield curve model parameters

In his research Svensson (1994)⁵ approaches the yield curve modeling by defining the forward yield curve⁶. By integrating the forward yield curve equation and by grouping the β parameters it is possible to obtain the following zero-coupon yield curve equation (Zdravkovic 2010, 28):

$$Z_n = \beta_1 + \beta_2 \left[\frac{1 - e^{-\lambda_1 n}}{\lambda_1 n} \right] + \beta_3 \left[\frac{1 - e^{-\lambda_1 n}}{\lambda_1 n} - e^{-\lambda_1 n} \right] + \beta_4 \left[\frac{1 - e^{-\lambda_2 n}}{\lambda_2 n} - e^{-\lambda_2 n} \right]. \quad (5)$$

If the above equation (5) is compared to the equation (2) it is easy to notice that the Svensson model is an extension of the Nelson-Siegel model. The extension is related to the additional β and λ parameters (β_4 and λ_2) in the model. The expression in the parenthesis next to the β_4 represents the parameter (factor) loading just like in the case of β_2 and β_3 . It contains the additional λ parameter which determines the rate of decay

⁵ Svensson and Söderlind (1997) published an extended version of the original research. Sometimes the model is therefore referred to as the Svensson Soderlind model.

⁶ Nelson and Siegel (1987) have done the same.

of the β_4 as the maturity increases. The two additional parameters allow greater flexibility at the short end of the yield curve enabling the model to capture the double humped (S shaped) form of the yield curve. Other factors and expressions are just the same as in the Nelson-Siegel model and therefore have the same meaning.

Due to the above stated similarities between the Nelson-Siegel and Svensson models it is clear that the equation (5) can be rewritten in the form of the regression equation just like the equation (2) in the section 2.1. The model thus takes on the functional form given by equation (6):

$$y_t(\tau) = G * \beta_t + e_t, \quad (6)$$

where $y_t(\tau)$ represents a vector of yields observed in the moment t for T maturities in the vector τ , error term is given by $e \sim N(0, R)$, β_t collects the yield curve factors (referring to the level, slope and the two measures of curvature) and G represents factor loadings matrix (Nyholm 2008, 73):

$$G = \begin{bmatrix} 1 & \frac{1 - \exp(-\lambda_1 \tau_1)}{\lambda_1 \tau_1} & \frac{1 - \exp(-\lambda_1 \tau_1)}{\lambda_1 \tau_1} - \exp(-\lambda_1 \tau_1) & \frac{1 - \exp(-\lambda_2 \tau_1)}{\lambda_2 \tau_1} - \exp(-\lambda_2 \tau_1) \\ 1 & \frac{1 - \exp(-\lambda_1 \tau_2)}{\lambda_1 \tau_2} & \frac{1 - \exp(-\lambda_1 \tau_2)}{\lambda_1 \tau_2} - \exp(-\lambda_1 \tau_2) & \frac{1 - \exp(-\lambda_2 \tau_2)}{\lambda_2 \tau_2} - \exp(-\lambda_2 \tau_2) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{1 - \exp(-\lambda_1 \tau_T)}{\lambda_1 \tau_T} & \frac{1 - \exp(-\lambda_1 \tau_T)}{\lambda_1 \tau_T} - \exp(-\lambda_1 \tau_T) & \frac{1 - \exp(-\lambda_2 \tau_T)}{\lambda_2 \tau_T} - \exp(-\lambda_2 \tau_T) \end{bmatrix}. \quad (7)$$

As already explained in the section 2.1. the expression given by equation (7) can be linearized if the parameters λ_1 and λ_2 are treated as constants rather than being variables that need to be estimated. The regression equation given by (6) is therefore solved for different values of λ_1 and λ_2 parameters chosen arbitrarily from a predetermined interval. The optimal value of parameters is then finally determined as the one which minimizes the sum of squared residuals between the observed and yields estimated by the model just as described in the case of the Nelson-Siegel model.

Also, once the Svensson model parameters have been estimated they can be used to estimate the whole term structure (i.e. estimate any yield for any given maturity) for any given point in time in the analyzed sample.

2. DATA

Since the Croatian government has only issued 15 bonds in the past 20 years yield curve was estimated using both data on government bonds and treasury bills. The available instruments are sometimes issued as pure kuna instruments and sometimes as instruments with foreign currency (euro) clause. Therefore two separate samples were initially formed regarding this distinctive characteristic. Mid yields were collected via Bloomberg on all government bonds. For treasury bills Ministry of finance data was used which refers to the yields achieved at regular auctions. Since there is little trading activity when it comes to treasury bills this was the best source of data possible. Monthly averages were calculated for all the instruments based on collected yields which enabled yield curve estimation on monthly basis.

After looking at the collected data for two samples it became clear that there were a lot of months with no trading activity or worse with no instruments to be traded which

resulted in only one or two data points available per month. To be exact for sample referring to the pure kuna instruments 4 or more data points have been available continuously only starting from April 2006. For the foreign currency clause sample 4 or more data points have been available from March 2004. Market data have been collected for both samples up to June 2011. Thus samples range from April 2006 to June 2011 (63 observations) for the pure kuna sample and from March 2004 to June 2011 (88 observations) for the foreign currency clause sample.

As financial markets convention is to report yields on government bonds on yield to maturity basis, zero-coupon yields had to be calculated using the bootstrapping technique for both samples. This was carried out in MATLAB software using MATLAB's "zbtyield" function.

3. RESEARCH FINDINGS

Parameters of the Nelson Siegel and Svensson models were estimated for both samples of the collected data by using a MATLAB code which carried out calculations specified by the equations (3) and (6). Estimated parameters were then used to estimate the term structure (the yield curve) for every observation in both data samples. For each observation yields were estimated for the following maturities: 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months.

Two problems appeared in the process. First, it turned out that standard errors of the parameter estimators were approaching infinity for some observations. This appears to be the case in both data samples for both models when parameters are estimated based on only 4 data points for a given observation. Therefore it seems that at least 5 data points are necessary to estimate parameters for both models properly. Due to the described problem the sample referring to the pure kuna instruments was reduced by 1 observation to 62 observations, while the sample referring to the foreign currency clause instruments had to be reduced by 11 observations to 77 observations. Second, in some instances estimated model parameters resulted in unusually low or high yield estimates at the short end of the yield curve (maturities up to 1 year but mostly regarding maturities referring to 3 and 6 months). This kind of a problem also appeared in both data samples whenever an observation lacked data on the short end of the yield curve. Obviously when this is the case the models adjust themselves too much to the available data leading to distortion in the estimates of the short end of the curve. As this problem was also more evident in the foreign currency clause instruments sample graphs in Figures 2 and 4 were adjusted to show only the maturity spectrum ranging from 2 to 10 years.

Regarding the second problem one can notice that in the case of the Svensson model and the foreign currency clause instruments sample even after trimming the maturity spectrum there are a lot of spikes at the short end of the yield curve (see Figure 4). Generally, if Figures 1–4 are compared it can easily be noticed that the Nelson-Siegel model produces smoother yield curve estimates with fewer upward and downward pointing spikes than the Svensson model. Therefore it can be concluded that the Svensson model seems to be overparameterized in the illiquid and undeveloped financial market. The model adjusts itself too much to the few available data points per observation due to additional parameters turning its increased flexibility into a disadvantage in such a scenario.

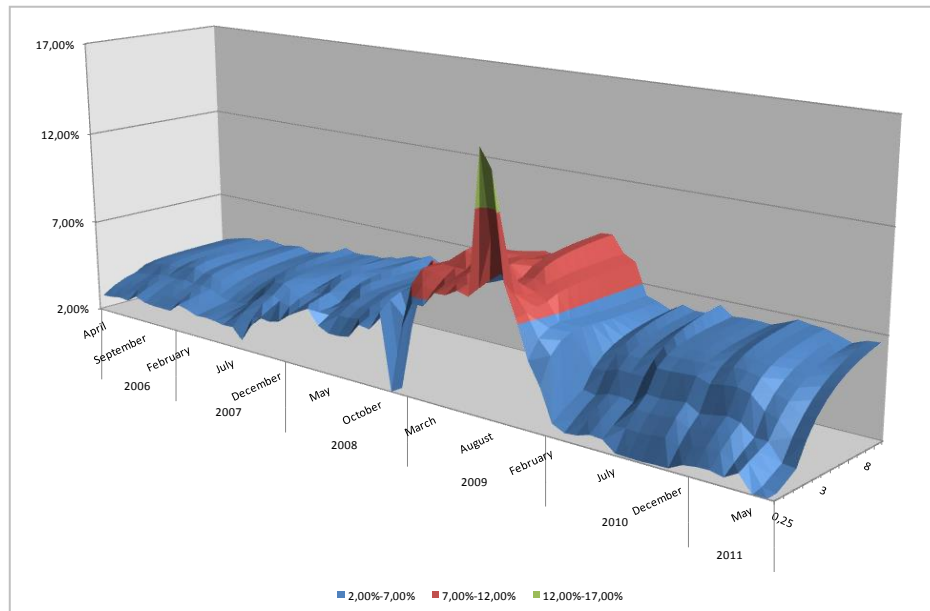


Figure 1. Yield curve evolution estimated by Nelson Siegel model (pure kuna instruments) (Zoricic 2012)

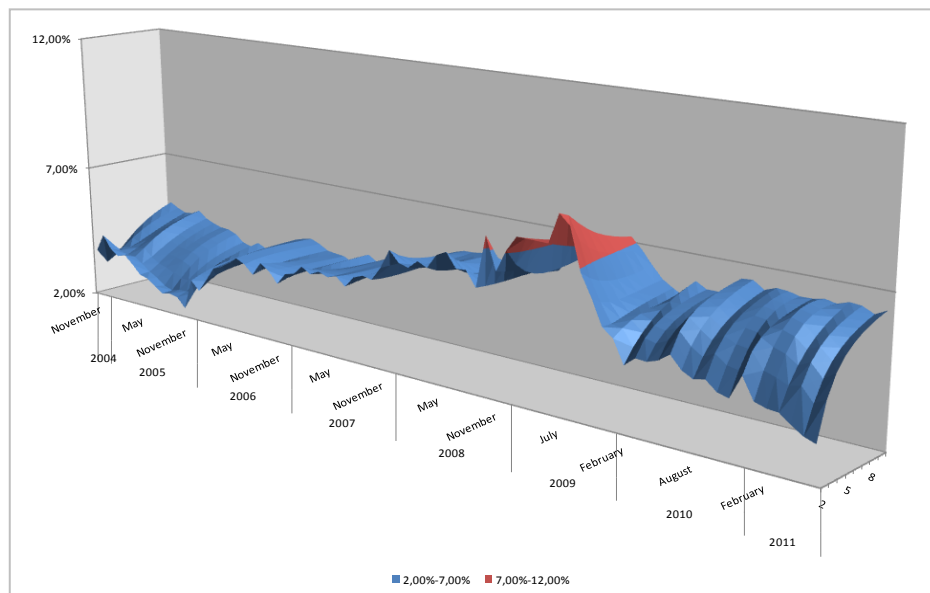


Figure 2. Yield curve evolution estimated by Nelson Siegel model (foreign currency clause instruments) (Zoricic 2012)

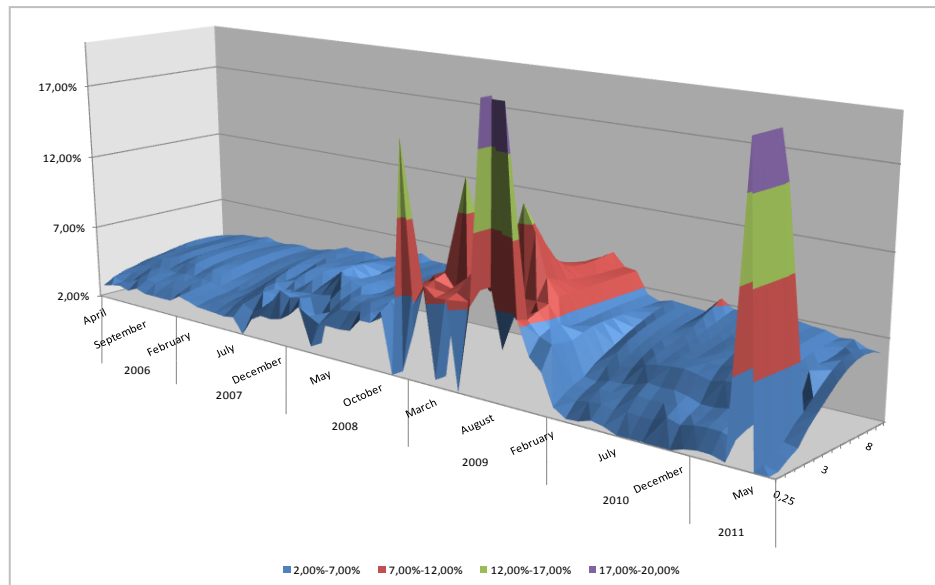


Figure 3. Yield curve evolution estimated by Svensson model (pure kuna instruments) (Zoricic 2012)

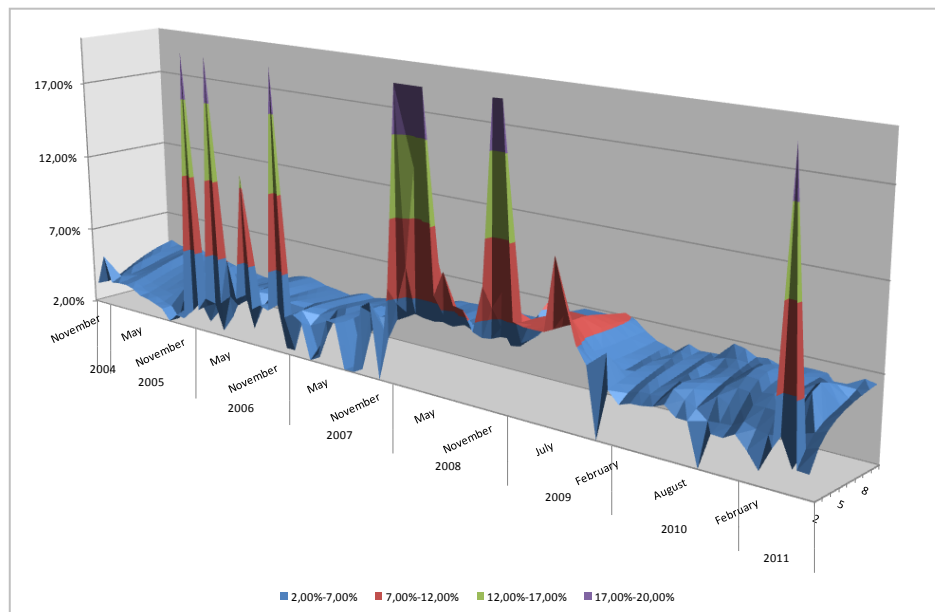


Figure 4. Yield curve evolution estimated by Svensson model (foreign currency clause instruments) (Zoricic 2012)

Regardless of the encountered problems importance of the estimated yield curves is at least threefold. Firstly, the Nelson-Siegel yield curve model is preferred over the more complex Svensson model like in the case of the Slovenian financial market as shown by Grum (2006). Secondly, yield curves for both samples and models seem to exhibit changes in their shapes in line with the macroeconomic theory (inverted at the beginning of the world financial crisis, steeply upward sloping in the lasting recession, mildly upward sloping during economic expansion, etc. — see Figures 1–4) which shows that the examined yield curve models are a desirable tool when trying to form a clear picture regarding the yield curve shape and its dynamics. Financial market's illiquidity does not seem to diminish or distort this significantly. Thirdly, unobservable market yields can be extrapolated and interpolated when using a yield curve model. In undeveloped financial markets this is particularly useful as it enables continuous availability of yields of arbitrarily chosen maturities.

The application of the examined yield curve models in such an environment however lacks the potential to serve in the field of risk management and derivatives but allows valuable insight regarding market expectations. The latter could even be used to some extent when it comes to bond valuation and pricing.

CONCLUSION

The paper deals with the parametric approach to the yield curve modeling in an illiquid and undeveloped financial market. It shows that it is possible to use the Nelson-Siegel and Svensson yield curve models in such an environment to model the yield curve in spite of the challenges concerning primarily the available market data.

The research results seem to suggest that a minimum of 5 data points need to be available for every observation in the sample in order to estimate the Nelson-Siegel and Svensson yield curve models parameters properly. Also data should cover as much of the maturity spectrum as possible, especially at the short end of the curve to prevent distortions in the estimates caused by the lack of data.

Also, given the illiquidity and the underdevelopment of the Croatian financial market in the analyzed period the Nelson-Siegel model is preferred over the Svensson model. Due to the additional parameters the Svensson model suffers from overparameterization in the Croatian financial market. It adjusts itself too much to the available sample data which leads to the yield curve estimates that are not smooth and are often distorted by upward and downward pointing spikes at the short end.

Furthermore the estimated yield curves for the analyzed data samples seem to show that the yield curve evolution is in line with the macroeconomic theory over the analyzed period. Therefore it is possible to apply the Nelson-Siegel and Svensson models with regard to collecting information on market expectations. Neither the analyzed yield curve models nor the available data support the application of estimated yield curves in the field of risk management and derivatives.

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